Basic structures

Sets

* **Definition of sets:** A set is a collection of well defined object is called a set.

Ex: Odd number less than 10
i.e. 1, 3, 5, 7, 9.

Note: 1) Sets are usually denoted by capital letters i.e. A, B, C, X, Y, Z

2) The elements of set is represented by small letters i.e. a, b, c, x, y, z & p, q, r etc

3) If x is a element of set A then we write $x \in A$

If x is not a element of set A then we write $x \notin A$.

4) If the elements are repeated they are written once

ex: $A = \{1, 2, 2, 3, 4, 3\}$

$A = \{1, 2, 3, 4\}$

* There are two methods to represent a set

1) Roaster form. 2) Set builder form.

1. **Roaster form:** In these method set can be describe by a listing of elements which are seperated by ',' (comma)

ex. $A = \{1, 2, 3, ----\}$

2) Set builder Form :

In these method all the elements of the set possess single common which is not possesed. by any element outside the set.

ex: $R = \{ (x ; x$ is a natural number $\}$

Types of sets :

1. Empty set / void set / null set :

A set not having number of an element is called the empty set.

It is denoted by ϕ OR $\{ \}$

OR A set which does not contain any element is called the empty set

$A = \{x : 1 < x < 2$, x is a natural number$\}$

$\therefore$ $A = \{ \}$ or $A = \phi$

Here no natural number lies between 1 and 2.

2 singleton set : A set containing only one element

ex. $A = \{x : 1 < x < 3 \quad x \in N\}$

$\therefore$ $A = \{2\}$

3. Finite set : A set A is said to be finite set if element of set A is countable as. $\{1, 2, 3, 4\}$

4. Infinite set: A set containing uncountable number of elements are called Infinite set.

Ex: N = {set of natural number}
N = {1, 2, 3, - - - - -}

5. Equal set: When two sets have exactly same elements such sets are called equal set.

ex A = {1, 2, 3, 4} and B = {1, 4, 3, 2}
then A = B OR B = A.

6. Subset: A set A is said to be subset of B if every element of set A is also an elements of set B is called subset
: It is denoted by $A \subset B$

ex: A = {1, 2, 3} and B = {1, 2, 3, 4, 5}
$A \subset B$

Note: 1) Empty set is subset of every set i.e. $\phi \subseteq A$
2) every set is subset of itself.

7. Power set: The collection of all subset of set A is called a power set of A
It is denoted by P(A)

ex: A = {1, 2, 3}
P(A) = {{1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} ϕ}

P(A)

Ex. A : {a, b}
p(A) = {{a}, {b}, {a, b} ∅ }

Note: To find total number of subset.
2m. n [P(A)] = 2^n where n(A) = n
Ex: A = {a, b}
p(A) = {{a}, {b}, {a, b} ∅ }
n [p(A)] = 2^n
= 2^2
= 4

8) Cardinality of a set : Total number of elements in a set A is called cardinality of set.
∴ A it is denoted by (A) OR n(A).
then its cardinality in n(A) = 3
{1, 2, 3}

9. Universal set : A universal set is which contain all the elements of other set including its own elements.
It is usually denoted by "U"
Ex. A = {1, 2, 3, 4}, B = {3, 4, 5}, C = {5, 6, 7}
∴ U = {1, 2, 3, 4, 5, 6, 7}

1. Write the power set of A = {1, 2, 3, 4}.
Sol. P(A) = {{1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}

{2,3}, {3,4}, {2,4} {1,2,3}, {2,3,4},
{1,3,4}. {1,2,4} {1,2,3,4}, ∅}

Question: If A = {a, b, c} then find power set of A hence find cardinality of p(A).

Imp 2023 1st sem

Sol: A = {a, b, c}

p(A) = {{a}, {b}, {c}, {a, b}, {b, c}, {a, c}
{a, b, c} ∅ }

$n[p(A)] = 2^n$

$= 2^3$

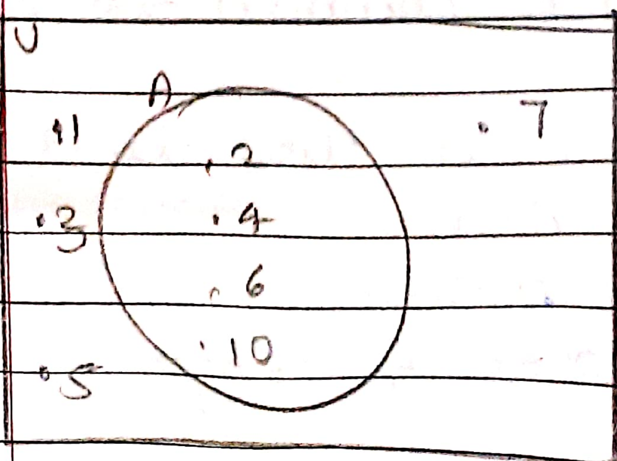
$= 8.$

Venn diagram: Most of the relationship between sets can be represented by means of diagram which are known as venn diagramm.

These diagram consist of rectangle and closed curves usually circles.

The universal set is represented usually by a rectangle and its subsets by circles.

In venn diagram the elements of the sets are written in their respective circles



U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {1, 3, 5, 7}

Imp SM 2022

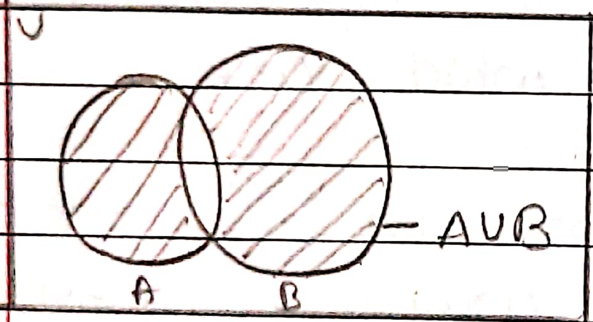
Operations on sets:

1) **Union of sets:** The union of two sets A and B is the set of which consist of all those elements which are either in A or in B.

In symbol we write

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

The union of two sets can be represented by a venn diagram.



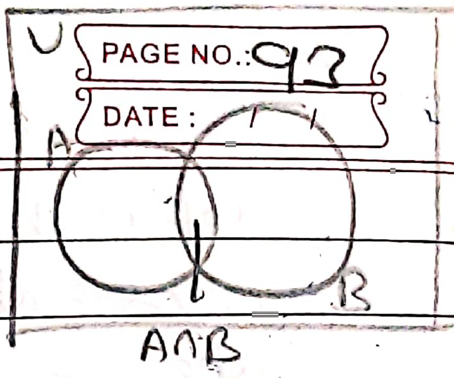
Ex: $A = \{2, 3, 5\}$, $B = \{5, 6, 7\}$ then A∪B is
i.e. $A \cup B = \{2, 3, 5, 6, 7\}$

2) **Intersection of sets:** The intersection of sets A and B is the set of all elements which are common to both A and B.

The symbol ∩ is used to denote intersection ∴ $A \cap B$.
Symbolically we write

$A \cap B = \{x : x \in A \text{ \& } x \in B\}$

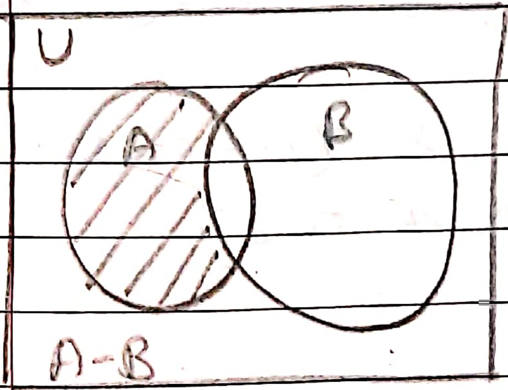
Ex: A = {a, b, c, d} B = {c, d, e}
A∩B = {c, d}



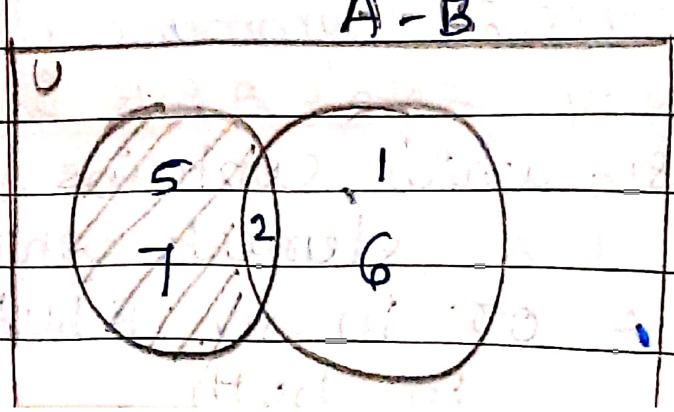
Difference of sets:

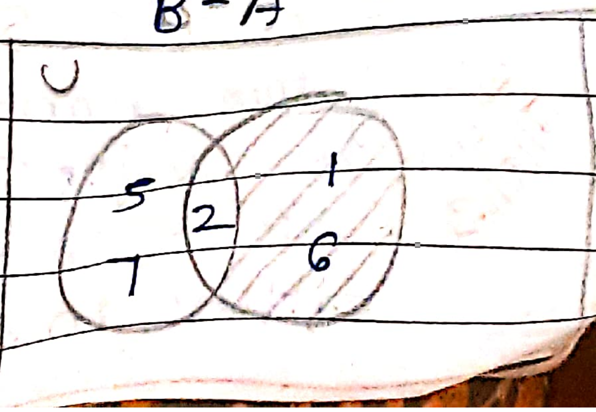
— The difference of sets A and B in this order is the set of elements which belong to A but not to B.
Symbolically we write "A − B" Read as A minus B.

— The difference of two sets A and B can be represented by venn diagram as shown in below.



Ex: A = {2, 5, 7} and B = {1, 2, 6} Find
Imp 2m 8/12 2022 ① A−B. ② B−A & draw its venn diagram.
A − B = {5, 7}
B − A = {1, 6}





Complement of set:

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the element of A.
Symbolically we write A' or A^c to denote the complement of A with respecting to U.
Thus $A' = \{x : x \in U \text{ and } x \notin A\}$
obviously $A' = U - A$.

ex: If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{1, 2, 3, 4\}$
$B = \{2, 4, 6, 8\}$. $C = \{3, 4, 5, 6\}$ then
find 1) A' 2) B', 3) C'

sol^n. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{1, 2, 3, 4\}$
$B = \{2, 4, 6, 8\}$ $C = \{3, 4, 5, 6\}$
$A' = U - A$
∴ 1) $A' = \{5, 6, 7, 8, 9\}$
$B' = U - B$
2) $B' = \{1, 3, 5, 7, 9\}$
$C = U - C$
3) $C' = \{1, 2, 7, 8, 9\}$

Symmetric Difference: Symmetric difference of two non-empty sets A & B is denoted by Def^n: The set which contains the
$A \Delta B = (A-B) \cup (B-A)$ elements which are either in set A or in set B but not in both

IMP 20/3/2023

Ex $A = \{2, 5, 7\}$ and $B = \{1, 2, 6\}$ Find $A \Delta B$

Soln Given $A = \{2, 5, 7\}$ $B = \{1, 2, 6\}$

then

$A \Delta B = (A-B) \cup (B-A)$

$A - B = \{5, 7\}$

$B - A = \{1, 6\}$

then

$A \Delta B = (A-B) \cup (B-A)$

$= \{5, 7\} \cup \{1, 6\}$

$= \{1, 5, 6, 7\}$

Examples.

Imp 2022 2m 1) If $A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6, 7, 8\}$ Find the i) $A \cup B$ ii) $A \cap B$

Soln $A = \{1, 2, 3, 4\}$ $B = \{4, 5, 6, 7, 8\}$

Then

i) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$

ii) $A \cap B = \{4\}$

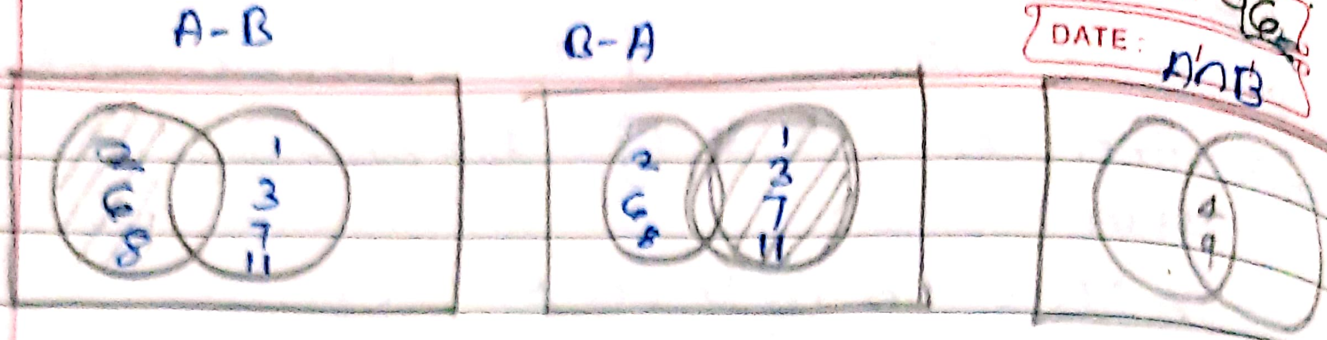
Imp 4M 2) Determine the set A and B given that $A-B = \{1, 3, 7, 11\}$, $B-A = \{2, 6, 8\}$ & $A \cap B = \{4, 9\}$

2021 Soln We have $A = (A \cap B) \cup (A-B)$.

$A \cap B = \{4, 9\} \cup \{1, 3, 7, 11\}$ $A = \{1, 3, 7, 11, 4, 9\}$

$B = (A \cap B) \cup (B-A) = \{4, 9\} \cup \{2, 6, 8\}$

$B = \{4, 9, 2, 6, 8\}$



3) If A = {1, 2, 3, 4} B = {3, 4, 5, 6} and C = {5, 6, 7, 8} D = {7, 8, 9, 10} find.
1) A∪B 2) B∪C 3) B∪D 4) A∪B∪C
5) A∩B∩C

Soln A = {1, 2, 3, 4} B = {3, 4, 5, 6}
C = {5, 6, 7, 8} D = {7, 8, 9, 10}

1) A∪B = {1, 2, 3, 4, 5, 6}

2) ii) B∪C = {3, 4, 5, 6} ∪ {5, 6, 7, 8}
= B∪C = {3, 4, 5, 6, 7, 8}

3) iii) B∪D = {3, 4, 5, 6} ∪ {7, 8, 9, 10}
= B∪D = {3, 4, 5, 6, 7, 8, 9, 10}

4) A∪B∪C = {1, 2, 3, 4} ∪ {3, 4, 5, 6} ∪ {5, 6, 7, 8}
= A∪B∪C = {1, 2, 3, 4, 5, 6, 7, 8}

5) A∩B∩C = {1, 2, 3, 4} ∩ {3, 4, 5, 6} ∩ {5, 6, 7, 8}
= A∩B∩C = {3, 4} ∩ {5, 6, 7, 8}
= A∩B∩C = { }.

4) If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} A
A = {1, 2, 3, 4}, B = {2, 4, 6, 8}
C = {3, 4, 5, 6} Find. 1) (A∪B)' 2) (B−C)'

Sol^n U = {1, 2, 3, 4, 5, 6, 7, 8, 9}
A = {1, 2, 3, 4} B = {2, 4, 6, 8}
C = {3, 4, 5, 6}

i) (A∪B) = {1, 2, 3, 4} ∪ {2, 4, 6, 8}
= {1, 2, 3, 4, 6, 8}
(A∪B)' = {5, 7, 9}

ii) (B−C) = {2, 8}
(B−C)' = {1, 3, 4, 5, 6, 7, 9}

5) U = {a, b, c, d, e, f, g, h} and
A = {a, b, c, d} B = {d, e, f} then find $\overline{A \cup B}$
$\overline{A} \cap \overline{B}$

Sol^n U = {a, b, c, d, e, f, g, h} A = {a, b, c, d}
B = {d, e, f}

i) A∪B = {a, b, c, d, e, f}
$\overline{A \cup B}$ = {g, h}

ii) $\overline{A} \cap \overline{B}$
$\overline{A}$ = {e, f, g, h} $\overline{B}$ = {a, b, c, g, h}

$\overline{A} \cap \overline{B}$ = {e, f, g, h} ∩ {a, b, c, g, h}
= {g, h}

(8) Determine the sets A & B given that

$A \cup B = \{1, 2, 4, 5, 7, 8, 9, 10\}$ $A \cap B = \{2, 4, 7\}$ $A - B = \{1, 8\}$

80|2 $A = (A \cap B) \cup (A - B) =$ ~~$\{1, 2, 4, 7, 8\}$~~

$A = \{2, 4, 7\} \cup \{1, 8\} = \{1, 2, 4, 7, 8\}$

$B = (A \cup B) - (A - B) =$ ~~$\{2, 4, 5, 7, 9, 10\}$~~

$B = \{1, 2, 4, 5, 7, 8, 9, 10\} - \{1, 8\}$

$B = \{2, 4, 5, 7, 9, 10\}$.

Imp With usual notation prove that associative

① i) $A \cup (B \cup C) = (A \cup B) \cup C$.

Sol/n $A \cup (B \cup C) = (A \cup B) \cup C$

Let $D = B \cup C$ and $E = A \cup B$

Take any $x \in A \cup (B \cup C)$

$x \in A \cup D$

i.e. $x \in A$ or $x \in D$

i.e. $x \in A$ or ($x \in B$ or $x \in C$)

i.e. ($x \in A$ or $x \in B$) or $x \in C$

i.e. ($x \in A$ or $x \in B$) or $x \in C$

i.e. $x \in E$ or $x \in C$

$x \in E \cup C = (A \cup B) \cup C$

Hence $A \cup (B \cup C) \subseteq (A \cup B) \cup C$

Similarly $(A \cup B) \cup C \subseteq A \cup (B \cup C)$

∴ $A \cup (B \cup C) = (A \cup B) \cup C$

ii) $A \cap (B \cap C) = (A \cap B) \cap C$ Prove that Associative law

Sol/n $A \cap (B \cap C) = (A \cap B) \cap C$

Let $D = B \cap C$ and $A \cap B = E$

Take any $x \in A \cap (B \cap C)$

$x \in A \cap D$

$x \in A$ and ($x \in B$ and $x \in C$)

($x \in A$ and $x \in B$) and $x \in C$

~~$x \in A$~~ $x \in E$ and $x \in C$

$x \in E \cap C = (A \cap B) \cap C$

Hence $A \cap (B \cap C) \subseteq (A \cap B) \cap C$

Similarly $(A \cap B) \cap C \subseteq A \cap (B \cap C)$

∴ $A \cap (B \cap C) = (A \cap B) \cap C$

law ② For any three sets A, B, C prove that distributive laws:

i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Sol^n $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Take any $x \in A \cap (B \cup C)$

$x \in A$ and ($x \in B$ or $x \in C$)

i.e ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)

i.e $x \in A \cap B$ or $x \in A \cap C$

i.e $x \in (A \cap B) \cup (A \cap C)$

∴ $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ ---- ①

Next take any $y \in (A \cap B) \cup (A \cap C)$ Then

($y \in A$ and $y \in B$) or $y \in A$ and $y \in C$

i.e $y \in A$ and ($y \in B$ or $y \in C$)

i.e $y \in A$ and $y \in (B \cup C)$

i.e $y \in A \cap (B \cup C)$

∴ $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ ---- ②

from ⓘ and ⓘⓘ it follows that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Sol^n $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Take any $A \cup (B \cap C)$

$x \in A$ or $x \in (B \cap C)$

$x \in A$ or ($x \in B$ and $x \in C$)

($x \in A$ or $x \in B$) and ($x \in A$ or $x \in C$)

$x \in A \cup B$ and $x \in A \cup C$

$x \in (A \cup B) \cap (A \cup C)$

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ —— (1)

Next take any $y \in (A \cup B) \cap (A \cup C)$

$(y \in A$ or $y \in B)$ and $y \in A$ or $y \in C$

$y \in A$ or $(y \in B$ and $y \in C)$

$y \in A$ or $y \in (B \cap C)$

$y \in A \cup (B \cap C)$

$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ —— (2)

From (1) and (2)

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(3) For any two sets A and B prove that the Demorgan's laws

i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Soln $\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A} \cap \overline{B} = \{x / x \in \overline{A}$ and $x \in \overline{B}\}$

$= \{x / x \notin A$ or $x \notin B\}$

$= \{x / x \notin (A \cup B)\}$

$= \overline{A \cup B}$

ii) $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Soln $\overline{A \cap B} = \overline{A} \cup \overline{B}$

$\overline{A} \cup \overline{B} = \{x / x \in \overline{A}$ or $x \in \overline{B}\}$

$= \{x / x \notin A$ and $x \notin B\}$

$= \{x / x \notin (A \cap B)\}$

$= \overline{A \cap B}$

Note: For any two sets A and B

1) If the words "total", "atleast", "either" or "how many comes" then it is denoted by $n(A \cup B)$

2) If "both" "and" comes then it is represented by $n(A \cap B)$

3) "neither" "Nor" comes then it is denoted by $n(\overline{A \cup B})$

4) If "Only A" comes then its is $n(A) - n(A \cap B)$

5) If a "survey" "group of" "sample of" comes then it is denoted by $n(U)$

Important result of union and intersection which can be used to solve the practicle problems.

1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2) $n(\bar{A}) = n(U) - n(A)$
3) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Examples:

Imp 4m

① In a class of 52 students 30 are studying AI, 28 are studying M.L. and 13 are studying both subject. How many in the class are studying atleast one of the subject? How many are studying neither of these subject.

Soln $n(U) = 52$ students.

$n(AI) = 30$ students are studying AI.

$n(ML) = 28$ students are studying ML.

$n(AI \cap ML) = 13$ are studying both the subjects. then

$n(\overline{A \cup B}) = ?$

$n(A \cup B) = ?$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= n(AI) + n(ML) - n(AI \cap ML)$

$= 30 + 28 - 13$

$= 58 - 13$

$n(A \cup B) = 45$

$n(\overline{A \cup B}) = n(U) - n(A \cup B)$

$= 52 - 45$

$n(\overline{A \cup B}) = 7$

② In a Sample of 100 Chips, 23 have a defect D_1, 26 have a defect of D_2, 30 have a defect D_3, 7 have of D_1 and D_2, 8 have defects of D_1 and D_3, 10 have

Imp 5m 2022

defects D_2 and D_3 and 3 have the three defects find the number of chips having.

i) At least one defect

ii) No defect:

Soln: $n(D_1) = 23$, $n(D_2) = 26$, $n(D_3) = 30$

$n(D_1 \cap D_2) = 7$, $n(D_1 \cap D_3) = 8$, $n(D_2 \cap D_3) = 10$

$n(D_1 \cap D_2 \cap D_3) = 3$

i) At least one defect

$$n(D_1 \cup D_2 \cup D_3) = n(D_1) + n(D_2) + n(D_3) - n(D_1 \cap D_2) - n(D_1 \cap D_3) - n(D_2 \cap D_3) - n(D_1 \cap D_2 \cap D_3)$$

$= 23 + 26 + 30 - 7 - 8 - 10 + 3$

$= 23 + 26 + 30 + 3 - 7 - 8 - 10$

$= 82 - 25$

$= 57$

$n(D_1 \cup D_2 \cup D_3) = 57$

ii) No defect:

$\overline{n(D_1 \cup D_2 \cup D_3)} = n(U) - n(D_1 \cup D_2 \cup D_3)$

$= 100 - 57$

$= 43$

Imp S(8) 2021 ③ In a survey of 260 college students the following data were obtained. 64 had taken C programming 94 had taken Python, 58 had taken Machine Learning, 28 had taken both C programming and Machine learning, 26 had taken both C programming and python and 22 had taken python & Machine learning and 14 had taken all three type of subjects. Determine how many of these had taken none of the three subject. Also represent it by venn diagram.

Sol^n $n(U) = 260$,

$n(C) = 64$ had taken C programming

$n(P) = 94$ had taken python

$n(ML) = 58$ had taken machine learning

$n(C \cap ML) = 28$ had taken both C programming and machine learning

$n(C \cap P) = 26$ had taken both C programming and python

$n(P \cap ML) = 22$ had taken both python and machine learning

$n(C \cap ML \cap P) = 14$ had taken all three types of subject.

$n(C \cup ML \cup P) = ?$

$n(\overline{C \cup ML \cup P}) = ?$

$n(C \cup ML \cup P) = n(C) + n(P) + n(ML) - n(C \cap ML)$
$- n(C \cap P) - n(P \cap ML) - n(C \cap ML \cap P)$

$= 64 + 94 + 58 - 28 - 26 - 22 + 14$

$= (64 + 94 + 58 + 14) - (28 + 26 + 22)$

$= 230 - 76$

$n(C \cup M \cup L \cup P) = 154$

$n(\overline{C \cup M \cup L \cup P}) = n(U) - n(C \cup M \cup L \cup P)$

$n(\overline{C \cup M \cup L \cup P}) = 260 - 154$

$= 106.$

4) If x and y are two sets such that $n(x) = 17$, $n(y) = 23$, and $(x \cup y) = 38$ find $n(x \cap y)$?

soln $n(x) = 17$, $n(y) = 23$, $(x \cup y) = 38$, ~~$n(x \cap y) = ?$~~

$n(x \cup y) = n(x) + n(y) - n(x \cap y)$

$38 = 17 + 23 - n(x \cap y)$

$38 = 40 - n(x \cap y)$

$n(x \cap y) = 40 - 38$

$n(x \cap y) = 2$

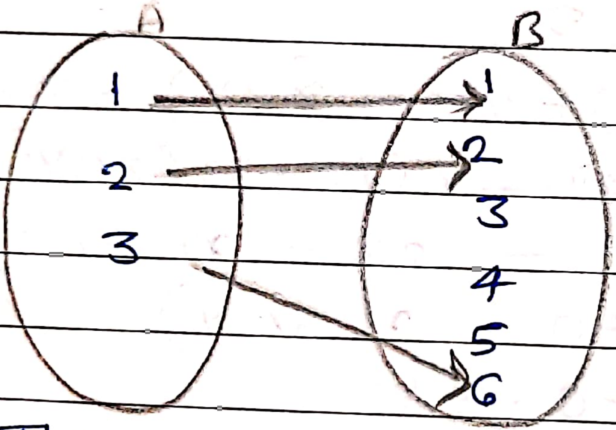
Function

2023 IMP 2nd sem

Defn: Relation R from set A to set B is said to be function if every element of set A should have one and only one element in set B.
It is denoted by $F: A \to B$

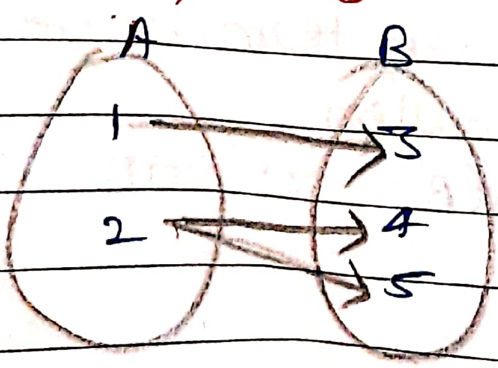
Ex: 1) $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5, 6\}$ Find the function are not?

Soln $A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4, 5, 6\}$
$F: A \to B$



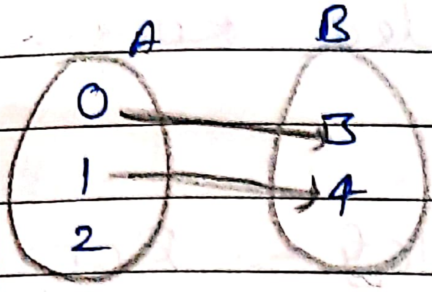
This is a function

2) Ex $A = \{1, 2\}$ $B = \{3, 4, 5\}$



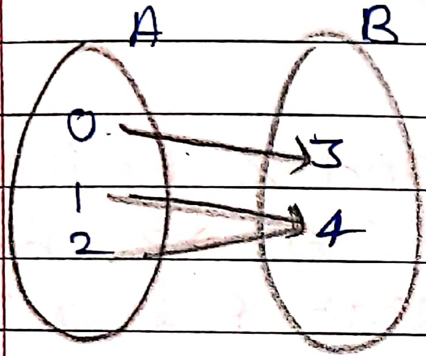
It is not a function because 2 is having more than one elements in set B.

3) $A = \{0, 1, 2\}$, $B = \{3, 4\}$



It is not function because 2 does not have element in set B.

4) $A = \{0, 1, 2\}$, $B = \{3, 4\}$



It is a function because every element of A having unique element in set B.

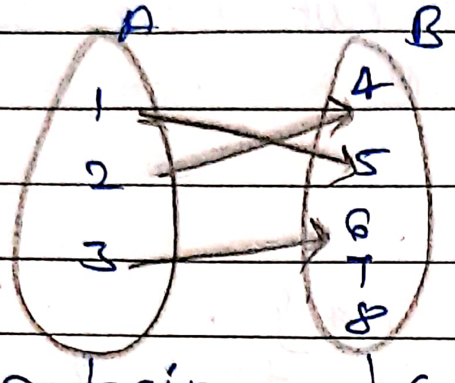
Note: 1) If f is a function from A to B is written as $f: A \rightarrow B$

2) If $f: A \rightarrow B$ and $(x, y) \in f$ then we write it as $y = f(x)$.

3) If $f: A \rightarrow B$ then set A is called Domain and set B is called Co-domain

4) If $f: A \rightarrow B$ and $(x, y) \in f$ then
x is called pre image of y under f
y is called image of x under f.

5) If $f : A \rightarrow B$ then the set of all image in B is called Range



$\{4, 5, 6\} \rightarrow$ Range.

Domain   Codomain

Question 1) Let $A = \{1, 2, 3, 4\}$ and Z be the set of integers defined $f : A \rightarrow Z$ by $f(x) = 2x+3$ show that f is a function from A to B. Also find out Range of function.

Soln Given $f : A \rightarrow Z$ where

$A = \{1, 2, 3, 4\}$

$f(x) = 2x + 3$

$f(1) = 2 \times 1 + 3 = 2 + 3 = 5$

$f(2) = 2 \times 2 + 3 = 4 + 3 = 7$

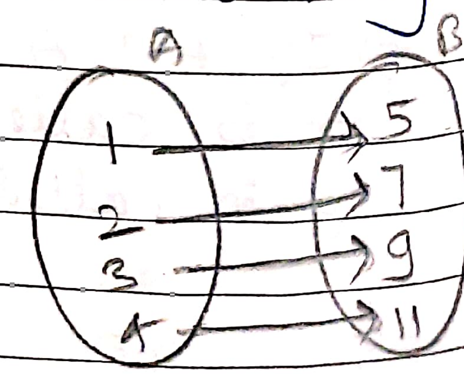
$f(3) = 2 \times 3 + 3 = 6 + 3 = 9$

$f(4) = 2 \times 4 + 3 = 8 + 3 = 11$

Since Every element of set A is having unique element

$\therefore$ It is a function

Range $= \{5, 7, 9, 11\}$



2) Let N be the set of natural numbers if $F: N \to N$ is defined by $F(x) = 2x-1$ then show that 'F' is a function and find Range

Sol^n Given $F: N \to N$.

Where $N = \{1, 2, 3, 4, 5, 6, - - - -\}$

$F(x) = 2x-1$

$F(1) = 2\times1-1 = 2-1 = 1$

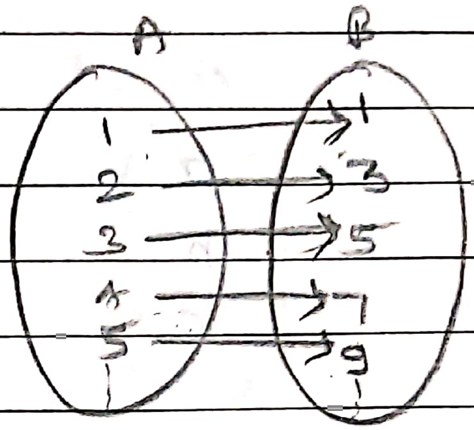
$F(2) = 2\times2-1 = 4-1 = 3$

$F(3) = 2\times3-1 = 6-1 = 5$

$F(4) = 2\times4-1 = 8-1 = 7$

$F(5) = 2\times5-1 = 10-1 = 9$

⋮



$F = \{(1,1), (2,3), (3,5), (4,7), (5,9)\}$

Since. Every element from domain is having unique element is having unique element is codomain

∴ It is a function

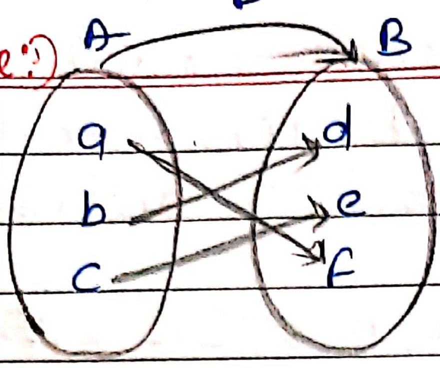
Range $R = \{1, 3, 5, 7, 9\}$

Types of Function:

2023 Imp 2m 1st sem 1) One - one function: A function $f: A \to B$ is said to be One-One function. If different element of set A should have different image of in B. under F.

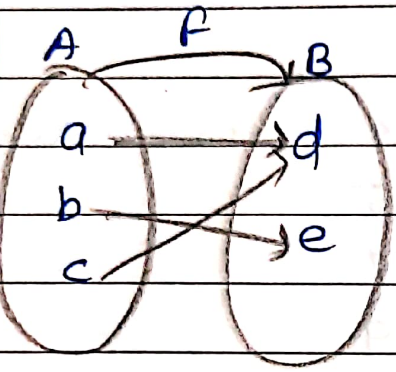
i.e. $a_1, a_2 \in A$ with $a_1 \neq a_2$ then $F(a_1) \neq F(a_2)$ OR equivalently that if whenever $f(a_1) = f(a_2)$ for $a_1, a_2 \in A$ then $a_1 = a_2$

Example :-
1)



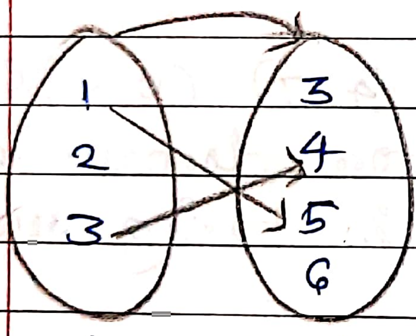
∴ It is one-one because different element of set A having different element of set B.

2)

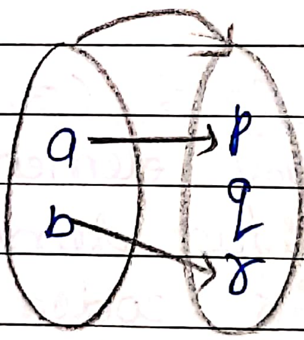


∴ It is not one-one because a and c having same elements in B i.e. d.

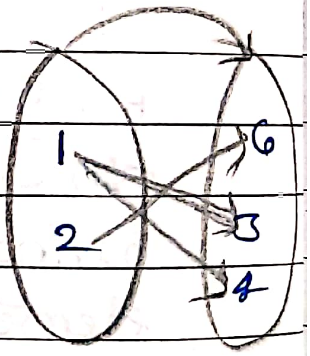
③



∴ It is not function



∴ It is a function



∴ It is not function

4) Let $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$ Consider the function $f : A \to B$ defined by $f(x) = 2x$ is one-one function

Sol/n $A = \{1, 2, 3\}$ $B = \{2, 4, 6\}$

$f(x) = 2x$

then

$f(1) = 2x = 2 \times 1 = 2$

$f(2) = 2 \times 2 = 2 \times 2 = 4$

$f(3) = 2 \times 3 = 2 \times 3 = 6$

$f = \{(1,2), (2,4), (3,6)\}$. Hence it is one-one

surjective

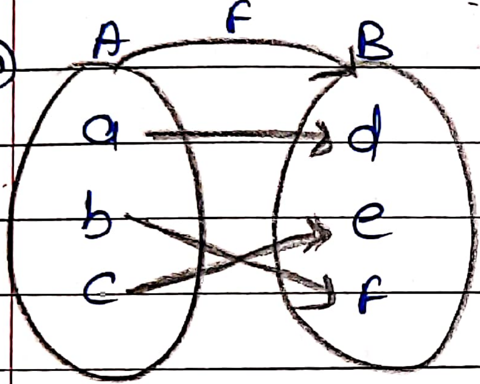
② **Onto Function:** A function $f: A \to B$ is said to be Onto if every element of set B should have atleast one element of set A.

Imp 2m 2023 1st sem

OR

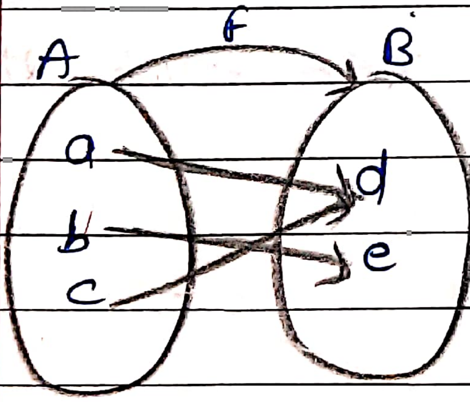
$f: A \to B$ is said to be onto function

Ex: 1)



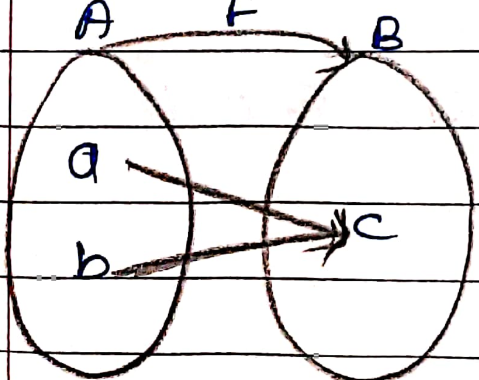
∴ It is Onto and One-One
$R = \{d, e, f\}$

2)



∴ It is onto because $\{d, e\} = B$
but not one-one

3



∴ It is onto but not One-One

4m

Question: If $f: R \to R$ defined by $f(x) = 2x+4$ then show that $f(x)$ is one-one and onto

Sol/n $f: R \to R$

for One-One

$f(x_1) = f(x_2)$

$2x_1 + 4 = 2x_2 + 4$

$2x_1 + 4 - 4 = 2x_2$

$2x_1 = 2x_2$

$\frac{2}{2} x_1 = x_2$

$x_1 = x_2$

∴ $f(x)$ is One-One

for onto

$\forall y \in R$ there exist $x \in R$ such that $f(x) = y$

$2x + 4 = y$

$2x = y - 4$

$x = \frac{y-4}{2}$

$f(x)$ is onto

∴ $f(x)$ is both One-One & onto

∴ $f(x)$ is bijective

2) Let a function $f: R \to R$ be defined by $f(x) = x^2 + 1$. Determine the image of the following subsets of R.

i) $A_1 = \{2, 3\}$ ii) $A_2 = \{-2, 0, 3\}$
iii) $A_3 = [-6, 3]$ iv) $A_4 = (0, 1)$

Soln We have

i) $A_1 = \{2, 3\}$, $f: R \to R$, $f(x) = x^2+1$

$f(x) = x^2+1 = f(2) = 2^2+1 = 4+1 = 5$

$f(3) = 3^2+1 = 9+1 = 10$

$\therefore f(A_1) = \{5, 10\}$

ii) $A_2 = \{-2, 0, 3\}$

$= f(x) = x^2+1 = (-2)^2+1 = 4+1 = 5$

$f(0) = 0^2+1 = 1$

$f(3) = 3^2+1 = 9+1 = 10$

iii) $A_3 = [-6, 3]$

$= f(A_3) = \{f(x) \mid -6 \le x \le 3\}$

$= \{(x^2+1) \mid -6 \le x \le 3\}$

iv) $A_4 = (0, 1)$

$= f(A_4) = \{x \in R \mid 0 < x < 1\}$

$\therefore f(A_4) = \{f(x) \mid 0 < x < 1\}$

$= \{(x^2+1) \mid 0 < x < 1\}$

Imp 3) Let function $F: R \rightarrow R$ defined by $F(x) = 2x^3 + x + 1$, determine image of
2M the subset $A_1 = \{-1, 2\}$ of R

Sol^n Given $F: R \rightarrow R$, $A_1 = \{-1, 2\}$

$F(x) = 2x^3 + x + 1$

$F(A_1) = 2 \times (-1)^3 + (-1) + 1$

$F(-1) = 2 \times (-1) \div 1 + 1$

$F(-1) = -2 - 1 + 1$

$F(-1) = -3 + 1$

$F(-1) = -2$

$F(A_1) = 2x^3 + x + 1$

$F(2) = 2(2)^3 + 2 + 1$

$F(2) = 2 \times 8 + 2 + 1$

$F(2) = 16 + 2 + 1$

$F(2) = 19$

$\therefore F(A_1) = \{-2, 19\}$

4) Let $A = \{0, \pm 1, \pm 2, 3\}$ Consider the function $F: A \rightarrow R$ (where R is the set of all real number) defined by $F(x) = x^3 - 2x^2 + 3x + 1$ for $x \in A$ Find the range of F.

Sol^n We find that, $F(x) = x^3 - 2x^2 + 3x + 1$

$F(0) = 0^3 - 2 \times 0 + 3 \times 0 + 1$

$F(0) = 1$

$f(1) = 1^3 - 2\times(1)^2 + 3(1) + 1$

$f(1) = 1 - 2 + 3 + 1 = 5 - 2 = 3$

$f(2) = 2^3 - 2\times(2)^2 + 3\times2 + 1$

$f(2) = 8 - 2\times4 + 6 + 1$

$f(2) = 8 - 8 + 6 + 1 = 7$

$f(-2) = (-2)^3 - 2\times(-2)^2 + 3(-2) + 1$

$f(-2) = -8 - 2\times4 - 6 + 1$

$f(-2) = -8 - 8 - 6 + 1$

$f(-2) = -22 + 1$

$f(-2) = -21$

$f(3) = 3^3 - 2\times(3)^2 + 3\times3 + 1$

$f(3) = 27 - 2\times9 + 9 + 1$

$f(3) = 27 - 18 + 9 + 1$

$f(3) = 37 - 18$

$f(3) = 19$

Hence the range of f is

$f(A) = \{1, 3, 7, -21, 19\}$

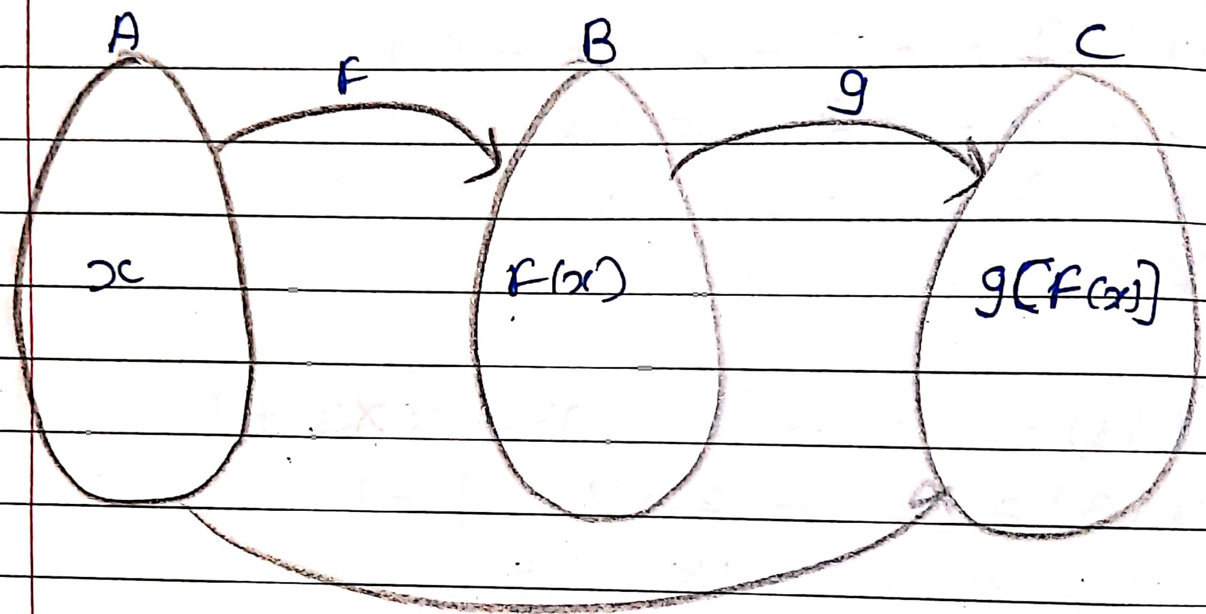
* composition of function :

The functions f : A → B and g : B → C

IMP 2M BCA Jorsen

The composition of these two functions is defined as the function g∘f : A → C with

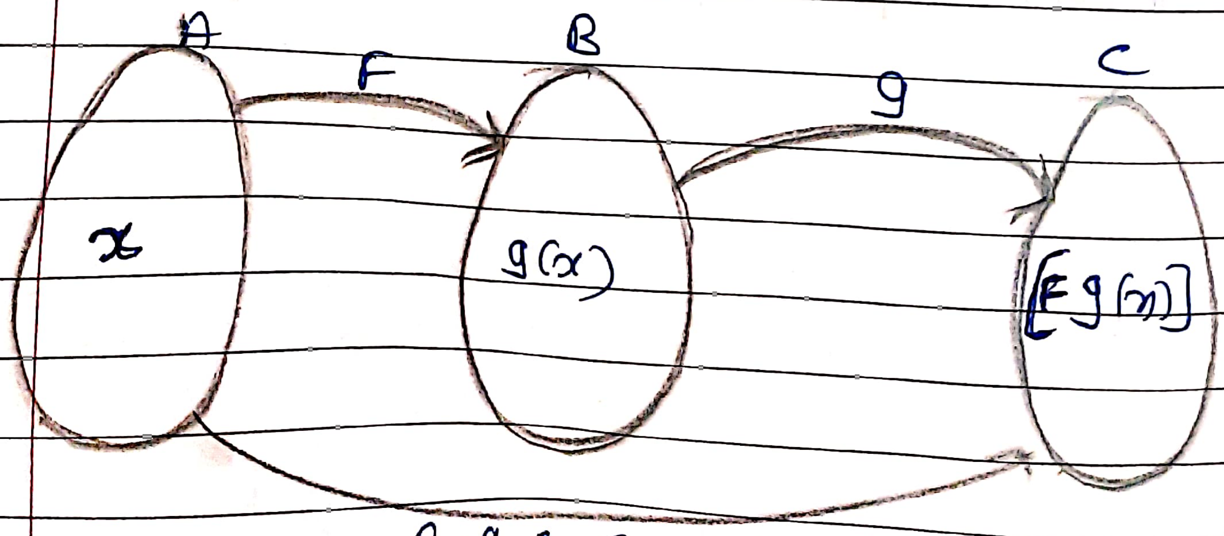
$$(g \circ f)(a) = g\{f(a)\} \text{ for all } a \in A$$



g∘f : A → C

A → C g∘f = [g f(x)]

C → A f∘g = [f g(x)]



f∘g : C → A

Imp

Eq. 2019 2013

Consider the function f and g defined by $f(x) = x^3$ and $g(x) = x^2+1$ find $g \circ f$

2013 Sol^n Here both f and g are defined on R. ∴ all the function $g \circ f$ are defined by R and we find that

Imp BCA 1st sem 2022

$(g \circ f)(x) = g\{f(x)\} = g(x^3) = (x^3)^2+1 = x^6+1$

2022, 2023

2013 1st sem

Let $A = \{1,2,3,4\}$ $B = \{a,b,c\}$ and $C = \{w,x,y,z\}$ with $f: A \to B$ and $g: B \to C$ given by
$f = \{(1,a),(2,a),(3,b)(4,c)\}$ and
$g = \{(a,x),(b,y),(c,z)\}$ find $g \circ f$

Sol^n Given $A = \{1,2,3,4\}$, $B = \{a,b,c\}$
$C = \{w,x,y,z\}$ and $f: A \to B$ and $g: B \to C$

We find by using the definition of f and g that.

$(g \circ f)(1) = g\{f(1)\} = g(a) = x$

$(g \circ f)(2) = g\{f(2)\} = g(a) = x$

$(g \circ f)(3) = g\{f(3)\} = g(b) = y$

$(g \circ f)(4) = g\{f(4)\} = g(c) = z$

Thus $g \circ f = \{(1,x),(2,x),(3,y),(4,z)\}$

IMP 2019 (3) SEM

Let $f(n) = 3n^3 - 2n^2$ and $g(n) = 2n^2$ be defined for positive integers n. Then show that f and g have same order.

(91)

IMP BCA 1st SEM

4) Define composition of function. Let R be the set of real numbers. Define $f: R \to R$ and $g: R \to R$ by $f(x) = 3x-2$ & $g(x) = x^2+4$ find i) gof ii) fog.

Soln: $f(x) = 3x-2$, $g(x) = x^2+4$

then i) $gof = g[f(x)] = g[3x-2]$.

$= (3x-2)^2 + 4$

$= (3x)^2 + 2^2 - 2 \times 3x \times 2 + 4$

$= 9x^2 + 4 - 12x + 4$

$g \circ f = 9x^2 - 12x + 8$

$fog = f[g(x)] = f[x^2+4] = 3(x^2+4) - 2$

$= 3x^2 + 12 - 2$

$fog = 3x^2 + 10$

"Matrices"

Matrix: Matrix is an ordered of an rectangular array of number or function.

Note: 1) The number or function are called the element of an matrix.

2) Matrix it is denoted by capital letters A, B, C, D - - -

Zero-One Matrices and Directed graphs:

Consider the set $A = \{a_1, a_2 ---- a_m\}$ and $B = \{b_1, b_2, b_3 --- b_n\}$ of orders m and n respectively. Then $A \times B$ consists of all ordered pairs of the form (a_i, b_j) $1 \le i \le m$, $1 \le j \le n$. which are mn in number. Let R be a relation from A to B so that R is a subset of $A \times B$. Now let us put $m_{ij} = (a_i, b_j)$ and assign the values of 1 or 0 to m_{ij} according to the following Rule.

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

The $m \times n$ matrix formed by these m_{ij}'s is called the matrix of the relation R.

or the relation matrix for R.

It is denoted by M_R or $M(R)$.

Contains only 0 and 1 as its elements.

M(R) is also called the zero-one matrix for R.

Sequences And Summations

Sequence : A Sequence is a function whose domain is the set of integers greater than or equal to a particular integer n_0 usually this set is the set of natural number {1, 2, 3 - - -} or the set of whole numbers {0, 1, 2, 3, - - - -}

It denoted by a_n where n is the image of the integer. and call it a term of the sequence

An explicit formula or general formula for a sequence is a rule that shows how the values of a_k depends on k.

Ex: Define a sequence a_1, a_2, a_3 --- by the explicit formula.

$$a_k = \frac{k}{k+1} \text{ for all integers } k \geq 1$$

Sol: The first four terms of the sequence are.

$$a_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$a_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4}$$ and fourth term is $a_4 = \frac{4}{4+1} = \frac{4}{5}$

1) Write the first four terms of the sequence defined by the formula $b_j = 1+2^j$ For all integer $j \geq 0$

Sol^n. $b_0 = 1+2^0 = 1+1 = 2$

$b_1 = 1+2^1 = 1+2 = 3$

$b_2 = 1+2^2 = 1+4 = 5$

$b_3 = 1+2^3 = 1+8 = 9$

2) 5, 9, 13, 17.

3) 0, −5, −10, −15.

4) x+a, x+3a, x+5a

Summation: We can use the greek letter Σ to represent the summation of a set of terms from a sequence.
It is denoted by
What is the value of

1) $$\sum_{i=1}^{5} i = 1+2+3+4+5 = 15$$

What is the value of

2) $$\sum_{i=3}^{7}(2i+4) = 2(3)+4 + 2(4)+4 + 2\times5+4$$

$= 6+4 + 8+4 + 10+4 + \ldots$

$10 + 12 + 14 + 16$

$+ 18 = 70$

$$\sum_{j=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 =$$

$$= 1 + 4 + 9 + 16 + 25 = 55$$

Summation formulas:

1) $$\sum_{i=1}^{n} c = cn$$

2) $$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

3) $$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

4) $$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Example If A, B, C are finite sets prove the following extended Addition Principle

$|A \cup B \cup C| = |A| + |B| + |C| - ~~|A\cup~~ |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Imp 2019 5m

Soln $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

We have

$|A \cup B \cup C| = |A \cup D|$, where $D = B \cup C$

$= |A| + |D| - |A \cap D|$ By addition principle

$= |A| + |B \cup C| - |A \cap (B \cup C)|$

↓ addition principle

$= |A| + \{|B| + |C| - |B \cap C|\} - |(A \cap B) \cup (A \cap C)|$

↓ addition principle

$= |A| + |B| + |C| - |B \cap C| - \{|A \cap B| + |A \cap C| - |(A \cap B) \cap (A \cap C)|\}$

$= |A| + |B| + |C| - |B \cap C| - |A \cap B| - |A \cap C| + |A \cap B \cap C|$

This proves the required result

* Addition principle :

Suppose we consider the union of two sets A and B and wish to determine the order of A∪B which is obviously a finite set

– Since the element of A∪B consist of all elements which are in A or B or both A and B the number of element in A∪B is equal to the number of elements in A plus the number of elements in B minus the number of elements (if any) that are common to A and B

i.e. $|A \cup B| = |A| + |B| - |A \cap B|$

* Principle of inclusion – exclusion for two sets:

Which we can count the number of elements in the union of two finite sets is known as the Principle of inclusion – exclusion for two sets.

* Principle of disjunctive counting for two sets:

A and B are disjoint sets so that $A \cap B = \phi$ the addition principle stated above becomes

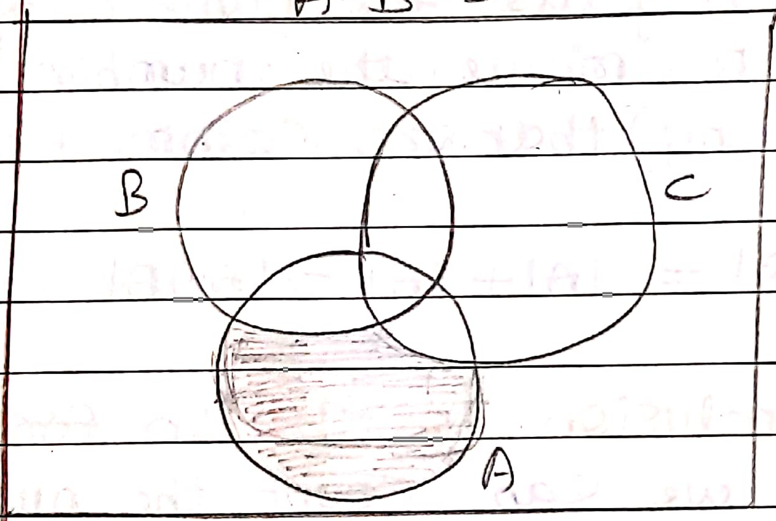
$$|A \cup B| = |A| + |B| - |\phi| = |A| + |B|$$

This is known as the principle of disjunctive counting for two sets.

2) If A, B, C are finite sets prove that $|A-B-C| = |A| - |A\cap B| - |A\cap C| + |A\cap B\cap C|$

Soln We first note that A−B−C is the set of elements that belong to A but not to B or C.

A−B−C



$\therefore |A-B-C| = |A\cup B\cup C| - |B\cup C|$

$= |A| + |B| + |C| - |A\cap B| - |B\cap C| + |A\cap B\cap C|$
$- (|B| + |C| - |B\cap C|)$ on using addition principle

$= |A| - |A\cap B| - |A\cap C| + |A\cap B\cap C|$