MARCH-2025 ANNUAL EXAMINATION – 01

SOLUTION OF QUESTION PAPER

PUC II Year MATHEMATICS

Time: 3 Hrs **[TOTAL QUESTIONS: 47]** **Max Marks: 80**

Instructions: (1) The question paper has five Parts namely A, B, C, D and E. Answer all the parts.
(2) Part-A has 15 multiple choice questions, 5 fill in the blank's questions of 1 mark each
(3) For Part-A questions, only the first written answers will e considered for awarding marks
(4) Use the graph sheet for the question on linear programming in Part-E.
(5) For questions having figure/graph, alternative questions are given at the end of question paper in separate section for visually challenged students

PART-A

I. Answer ALL the multiple choice questions : $(15 \times 1 = 15)$

1. **A relation R in the set A is called Reflexive relation if**
   a) $(a, a) \in R$ for all $a \in A$ b) $(a, a) \in R$ for atleast one $a \in A$
   c) $(a, b) \in R$ implies $(b, a) \in R$ d) $(a, b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$
   Ans: a) $(a, a) \in R$ for all $a \in A$

2. **The principal value of $sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is**
   a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$
   c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$
   Ans: c) $\frac{\pi}{4}$

3. **Match list I with List II**
4.

List I	List II
A) Domain of $sin^{-1}x$	**i) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$**
B) Range of $tan^{-1}x$	**ii) $[0, \pi]$**
C) Range of $cos^{-1}x$	**iii) $[-1, 1]$**

Choose the correct answer from the options given below:
a) A-i, B-ii, C-iii b) A-iii, B-ii, C-i
c) A-ii, B-i, C-iii d) A-iii, B-i, C-ii
Ans: d) A-iii, B-i, C-ii

5. **For 2×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = 2i - j$ then A is equal to**
   a) $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$
   c) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$
   Ans: b) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

6. **If A is a non-singular matrix of order 3×3, then $|adjA|$ is equal to**

a) $|A|$ b) $3|A|$

c) $|A|^3$ d) $|A|^2$

Ans: d) $|A|^2$

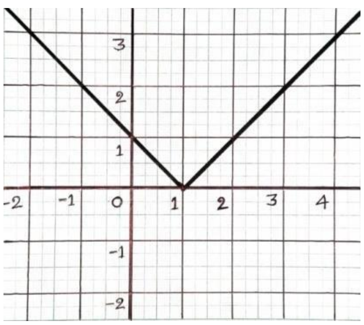
7. **If $f(x) = cos2x$ then $f'\left(\frac{\pi}{4}\right)$ is**

a) 2 b) -2

c) $\sqrt{2}$ d) $-\sqrt{2}$

Ans: b) -2

8. **For the figure given below, consider the following statements 1 and 2**



$y = f(x)$

Statement 1: The left hand derivative of $y = f(x)$ at $x = 1$ is -1

Statement 2: The function $y = f(x)$ is differentiable at $x = 1$

a) Statement 1 is true, Statement 2 is false b) Statement 1 is false, Statement 2 is true

c) Both Statements 1 and 2 are true d) Both Statements 1 and 2 are false

Ans: a) Statement 1 is true, Statement 2 is false

9. **The absolute maximum value of the function f given by $f(x) = x^3, x \in [-2, 2]$ is**

a) 2 b) 0

c) -2 d) 8

Ans: d) 8

10. **$\int e^x (sinx - cosx)dx$ is**

a) $-e^x cosx$ b) $e^x cosx$

c) $e^x sinx$ d) $e^x sin^2 x$

Ans: a) $-e^x cosx$

11. **The degree of the differential equation $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + e^{\frac{dy}{dx}} = 0$ is**

a) 1 b) 3

c) 2 d) $not\ defined$

Ans: d) $not\ defined$

12. **The direction cosines of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ are**

a) $\frac{1}{\sqrt{5}}, \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}},$ b) $\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}},$

c) $\frac{1}{6}, \frac{-1}{6}, \frac{2}{6}$ d) $\frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}},$

Ans: b) $\frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}},$

13. The angle between two vectors $\vec{a}$ and $\vec{b}$ with $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{6}$ is

a) $\frac{\pi}{6}$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{4}$
d) $\frac{\pi}{2}$

Ans: c) $\frac{\pi}{4}$

14. The equation of y-axis in space is

a) $x = 0, y = 0,$
b) $x = 0, z = 0$,
c) $y = 0, z = 0$,
d) $y = 0$

Ans: b) $x = 0, z = 0$,

15. If $P(A) = \frac{1}{2}, P(B/A) = \frac{2}{3}$ then $P(A \cap B)$ is

a) $\frac{1}{3}$
b) $\frac{1}{2}$
c) 1
d) $\frac{3}{5}$

Ans: a) $\frac{1}{3}$

16. Assertion [A] : For two events E and F if $P(E) = \frac{1}{5}$, $P(F) = \frac{1}{2}$ and $P(E/F) = \frac{1}{5}$ then E and F are independent events

Reason [R] : If E and F are two independent events then $P(F/E) = P(F)$

a) [A] is true but [R] is false
b) Both [A] and [R] are false
c) Both [A] and [R] are true
d) [A] is false but [R] is true

Ans: c) Both [A] and [R] are true

II. Fill in the blanks by choosing the appropriate answer from those give in the bracket : $(5 \times 1 = 5)$

$$(0,\ 2,\ 1,\ \frac{5}{9},\ -1,\ 6)$$

16. The value of $cos\left[sec^{-1}(2) - sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$ is ………………

Ans: 1

17. If $y = sin^{-1}(cosx)$ then $\frac{dy}{dx}$ =………………

Ans: -1

18. The value of $\int_7^{13} 1\, dx =$ ………………

Ans: 6

19. The projection of vector $\hat{\imath} + \hat{\jmath}$ on vector $\hat{\imath} - \hat{\jmath}$ is………………

Ans: 0

20. If $P(A \cap B) = \frac{4}{13}, P(B) = \frac{9}{13}$ $P(A'/B) =$ ………………

Ans: $\frac{5}{9}$

PART-B

III. Answer ANY SIX questions : $(\mathbf{6 \times 2 = 12})$

21. Find the equation of line joining $(1,2)$ *and* $(3,6)$ using determinants

Ans: Equation of line joining two points is given by

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x & y & 1 \end{vmatrix} = 0, \quad \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$\frac{1}{2}[1(6-y) - 2(3-x) + 1(3y-6x)] = 0,$$

$$6 - y - 6 + 2x + 3y - 6x = 0$$

$$-4x + 2y = 0$$

$$2y = 4x$$

$$y = 2x$$

22. If $\sqrt{x} + \sqrt{y} = \sqrt{10}$ then show that $\frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$

Ans: $\sqrt{x} + \sqrt{y} = \sqrt{10}$ diff w.r.t x

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\frac{dy}{dx} = 0$$

$$\frac{1}{2\sqrt{y}}\frac{dy}{dx} = -\frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\frac{dy}{dx} = -\sqrt{\frac{y}{x}},$$

$$\frac{dy}{dx} + \sqrt{\frac{y}{x}} = 0$$

23. A balloon, which always remains spherical has a variable radius. Find the rate at which its volume is increasing with the radius when the later is 10 cm

Ans: let r be the radius of spherical ballon and V be its volume

$$V = \frac{4}{3}\pi r^3$$

Rate of change of volume w.r.t radius

$$\frac{dV}{dr} = 4\pi r^2 \text{ when } r = 10cm$$

$$\frac{dV}{dr} = 4\pi(10)^2 = 400\pi\ cm^3/cm$$

Volume of balloon is increasing at the rate of $400\pi\ cm^3/cm$

24. Find the interval in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is decreasing

Ans: $f(x) = 4x^3 - 6x^2 - 72x + 30$

$f'(x) = 12x^2 - 12x - 72$

For solving the intervals, we have to take $f'(x) = 0$

$$12x^2 - 12x - 72 = 0$$

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3) = 0$$

$$x = -2 \text{ and } x = 3$$

Points $x = -2,3$ divides the real line into three disjoint intervals $(-\infty,-2)$, $(-2,3)$ and $(3,\infty)$

In the interval $(-\infty,-2)$ and $(3,\infty)$, function is increasing because $f'(x) > 0$

In the interval $(-2,3)$, function is decreasing because $f'(x) < 0$

25. Evaluate $\int cotx \cdot log(sinx)dx$.

Ans: $\int cotx \cdot log(sinx)dx = \int t\, dt$, put $t = log(sinx)$,

$= \frac{t^2}{2} + C$ $\quad dt = \frac{1}{sinx} \cdot cosxdx,\ dt = cotxdx$

$= \frac{(log(sinx))^2}{2} + C$

26. Verify that the function $y = acosx + bsinx$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$

Ans: $y = acosx + bsinx$ diff w.r.t x

$\frac{dy}{dx} = -asinx + bcosx$ again diff w.r.t x

$\frac{d^2y}{dx^2} = -acosx - bsinx$

$\frac{d^2y}{dx^2} = -(acosx + bsinx),\quad \frac{d^2y}{dx^2} = -y$

Then we get $\frac{d^2y}{dx^2} + y = 0$

27. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{c} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ then find the unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$

Ans: $\vec{d} = 2\vec{a} - \vec{b} + 3\vec{c}$

$\vec{d} = 2(\hat{\imath} + \hat{\jmath} + \hat{k}) - (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + 3(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

$\vec{d} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k} - 2\hat{\imath} + \hat{\jmath} - 3\hat{k} + 3\hat{\imath} - 6\hat{\jmath} + 3\hat{k}$

$\vec{d} = 3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$|\vec{d}| = \sqrt{9 + 9 + 4} = \sqrt{22}$

Now, The unit vector parallel to $2\vec{a} - \vec{b} + 3\vec{c}$ is

$\hat{d} = \frac{\vec{d}}{|\vec{d}|} = \frac{3\hat{\imath} - 3\hat{\jmath} + 2\hat{k}}{\sqrt{22}}$

$= \frac{3}{\sqrt{22}}\hat{\imath} - \frac{3}{\sqrt{22}}\hat{\jmath} + \frac{2}{\sqrt{22}}\hat{k}$

28. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular to each other, then find the value of k

Ans: $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$

If two lines are perpendicular then there scalar product is zero

$a_1b_1 + a_2b_2 + a_3b_3 = 0$

$(-3)(3k) + (2k)(1) + (2)(-5) = 0$

$-9k + 2k - 10 = 0$

$-7k = 10$

$k = -\frac{10}{7}$

29. An urn contains 10 black and 5 white balls. Two balls are drawn from the urn one after the other without replacement. What is the probability that both drawn balls are black?

Ans: E be the event first drawn ball is black

F be the event second drawn ball is black, if first is black

Now, $P(black\ ball\ in\ first\ draw)P(E) = \frac{10}{15}$

$P(black\ ball\ in\ second\ draw, if\ first\ is\ black\ ) = P(F/E) = \frac{9}{14}$

By multiplication rule of probability, we have

$P(E \cap F) = P(E) \cdot P(F/E)$

$= \frac{10}{15} \cdot \frac{9}{14} = \frac{3}{7}$

PART-C

IV. Answer ANY SIX questions: $(\mathbf{6 \times 3 = 18})$

30. Check whether the relation R in R of real numbers defined by $R = \{(a, b): a \leq b^3\}$ is reflexive, symmetric or transitive.

Ans: Reflexive: $\frac{1}{2} \leq (\frac{1}{2})^3$ $is\ not\ true$

$(\frac{1}{2}, \frac{1}{2}) \notin R$, ∴ R is not reflexive.

Symmetric: $(1,2) \in R \Rightarrow 1 \leq 2^3 \Rightarrow 2 \leq 1^3$ is not true,

∴ $(2,1) \notin R$, ∴ R is not symmetric.

Transitive: $(3,3/2)$ *and* $(3/2,6/5) \in R$,

$3 \leq (3/2)^3$ and $(3/2) \leq (6/5)^3$

But $3 \leq (6/5)^3$ is not true,

$(3,6/5) \notin R$, ∴ R is not transitive

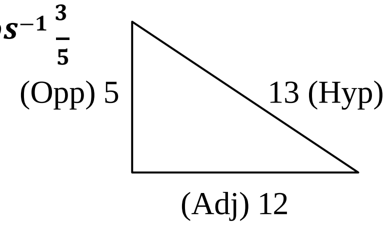
31. Prove that $\boldsymbol{tan^{-1}\frac{63}{16} = sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5}}$

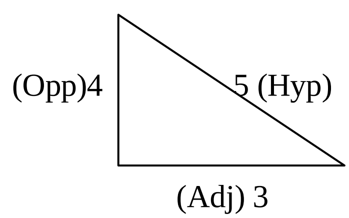
Ans: RHS $= sin^{-1}\frac{5}{13} + cos^{-1}\frac{3}{5}$

$= tan^{-1}\frac{5}{12} + tan^{-1}\frac{4}{3}$

$= tan^{-1}\left(\frac{(5/12)+(4/3)}{1-(5/12)(4/3)}\right)$

$= tan^{-1}\left(\frac{15+48}{36-20}\right) = tan^{-1}\frac{63}{16} = LHS$





32. Express the matrix $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$ as sum of Symmetric and Skew-symmetric matrices

Ans: $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$, $A^I = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

We know that $A = \frac{1}{2}(A + A^I) + \frac{1}{2}(A - A^I)$

$P = \frac{1}{2}(A + A^I)$

$= \frac{1}{2}\left[\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right]$

$= \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$Q = \frac{1}{2}(A - A^I)$

$= \frac{1}{2}\left[\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}\right]$

$= \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

Therefore $A = P + Q$

$$\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

33. Find $\frac{dy}{dx}$ if $x = a\left[cost + logtan\left(\frac{t}{2}\right)\right]$ and $y = asint$

Ans: $x = a\left[cost + logtan\left(\frac{t}{2}\right)\right]$

$$\frac{dx}{dt} = a\left[-sint + \frac{1}{tan\left(\frac{t}{2}\right)} \cdot sec^2\left(\frac{t}{2}\right) \cdot \frac{1}{2}\right] = a\left[-sint + \frac{1}{2sin\left(\frac{t}{2}\right)cos\left(\frac{t}{2}\right)}\right] = a\left[\frac{1-sin^2t}{sint}\right] = a\frac{cos^2t}{sint}$$

$y = asint$

$\frac{dy}{dt} = acost$

Now, dividing both the equations

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$= \frac{acost}{\frac{acos^2t}{sint}}$$

$$= tant$$

34. Find the two positive numbers x and y such that $x + y = 60$ and xy^3 is maximum

Ans: The two positive numbers $x = 60 - y$ and y

$$P(y) = (60 - y)y^3$$

$$P(y) = 60y^3 - y^4$$

$$P'(y) = 180y^2 - 4y^3$$

$$P''(y) = 360y - 12y^2$$

For the function is maximum or minimum $P'(x) = 0$

$$180y^2 - 4y^3 = 0$$

$$4y^2(45 - y) = 0$$

$$y = 0 \text{ or } y = 45$$

At $y = 0$, $P''(y) = 0$ therefore the function is neither maximum nor minimum

At $y = 45$, $P''(y) = -8100 < 0$ therefore xy^3 is maximum, then $x = 60 - 45 = 15$

The two positive numbers $x = 15$ and $y = 45$

35. Evaluate $\int \frac{2x}{x^2+3x+2} dx$

Ans: $\int \frac{2x}{x^2+3x+2} dx = \int \frac{2x}{(x+2)(x+1)} dx$

Let $\frac{2x}{(x+2)(x+1)} = \frac{A}{(x+2)} + \frac{B}{(x+1)}$ By using partial fraction

$$2x = A(x + 1) + B(x + 2)$$

Put $x = -2$ get $A = 4$ and put $x = -1$ get $B = -2$

$$\int \frac{2x}{(x+2)(x+1)} dx = \int \left[\frac{A}{(x+2)} + \frac{B}{(x+1)}\right] dx$$

$$= \int \left[\frac{4}{(x+2)} - \frac{2}{(x+1)}\right] dx$$

$$= \int \frac{4}{(x+2)} dx - \int \frac{2}{(x+1)} dx$$

$$= 4\log|x + 2| - 2log|x + 1| + C$$

36. Find the area of triangle ABC whose position vectors are of A, B and C are $\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $2\hat{\jmath} + \hat{k}$ and $\hat{\jmath} + 3\hat{k}$ respectively (MQP6)

Ans: $\overrightarrow{OA} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\overrightarrow{OB} = 2\hat{\jmath} + \hat{k}$

$\overrightarrow{OC} = \hat{\jmath} + 3\hat{k}$

Then $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\hat{\imath} + 3\hat{\jmath} - \hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 3 & -1 \\ -1 & 2 & 1 \end{vmatrix}$$

$\overrightarrow{AB} \times \overrightarrow{AC} = \hat{\imath}(3+2) - \hat{\jmath}(-1-1) + \hat{k}(-2+3)$

$\overrightarrow{AB} \times \overrightarrow{AC} = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$ w.k.t $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{25 + 4 + 1} = \sqrt{30}$

Area of triangle $= \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{30}\ sq\ units$

37. Derive the Equation of the line in space passing through a point and parallel to the vector in vector form

Ans: Let l be the equation of line which is passes through the point A with position vector $\vec{a}$

Let P be any arbitrary point on the line with position vector $\vec{r}$

Therefore $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{OP} = \vec{r}$

And the vector $\vec{b}$ is parallel to l

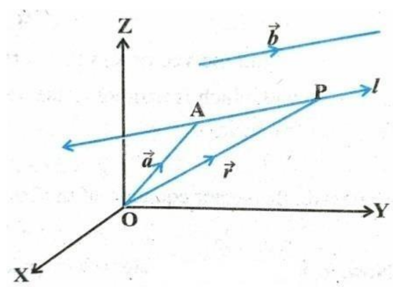
Therefore $\overrightarrow{AP}$ is parallel to the vector $\vec{b}$

$\overrightarrow{AP} = \lambda \vec{b}$

$\overrightarrow{OP} - \overrightarrow{OA} = \lambda \vec{b}$

$\vec{r} - \vec{a} = \lambda \vec{b}$

$\vec{r} = \vec{a} + \lambda \vec{b}$ (Vector form)



38. In two identical boxes, box I contains 2 gold coins, while box II contains one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

Ans: Let E_1 and E_2 be the events that boxes I and II are chosen respectively

$P(E_1) = \frac{1}{2}$ $P(E_2) = \frac{1}{2}$

Now, Let A be the event that the coin drawn is of gold

Draw a gold coin from box I $P(A/E_1) = \frac{2}{2} = 1$

Draw a gold coin from box II $P(A/E_2) = \frac{1}{2}$

The probability that the gold coin is drawn from the box I is $P(E_1/A)$

By Bayes' theorem, we have

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$$

$$= \frac{\frac{1}{2}\cdot 1}{\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{4}} = \frac{1}{2}\cdot\frac{4}{3} = \frac{2}{3}$$

PART-D

V. Answer ANY FOUR questions : $(4 \times 5 = 20)$

39. If $A = R - \{3\}$ and $B = R - \{1\}$ and $f: A \to B$ is a function defined by $f(x) = \frac{x-2}{x-3}$. Is f one-one and onto? Justify your answer.

Ans: $f: R - \{3\} \to R - \{1\}$ defined by $f(x) = \frac{x-2}{x-3}$

One-one: Consider $x_1, x_2 \in R - \{3\}$

then $f(x_1) = f(x_2)$,

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3},$$

$$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3),$$

$$x_1x_2 - 2x_2 - 3x_1 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6,$$

$$-2x_2 - 3x_1 = -2x_1 - 3x_2,$$

$$3x_1 - 2x_1 = 3x_2 - 2x_2,$$

$x_1 = x_2$, ∴ f is one-one

Onto: $y \in R - \{1\}$ there exists $x \in R - \{3\}$

such that $f(x) = y$,

$$\frac{x-2}{x-3} = y,$$

$$x - 2 = y(x - 3)$$

$$yx - x = 3y - 2,$$

$$x(y - 1) = 3y - 2,$$

$$x = \frac{3y-1}{y-1} \in R - \{3\},$$

$$f(x) = f\left(\frac{3y-1}{y-1}\right) = \frac{\left(\frac{3y-1}{y-1}\right)-2}{\left(\frac{3y-1}{y-1}\right)-3} = y,$$

∴ f is onto,

∴ f is both one-one and onto

40. If $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$ then verify that $(AB)' = B'A'$

Ans: $A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$ and $B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

Now, $AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$$

Now, $(AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$ ....(1)

Now, $B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

$$= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$ ……………(1)

From (1) and (2),

Therefore $(AB)' = B'A'$

41. Solve the following system of linear equation by matrix method

$$4x + 3y + 2z = 60$$
$$2x + 4y + 6z = 90$$
$$6x + 2y + 3z = 70$$

Ans: The system of equation is written as $AX = B$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$|A| = 50 \neq 0$ Therefore A is non singular

$$Cofactor\ of\ A = \begin{bmatrix} 0 & 30 & -20 \\ -5 & 0 & 10 \\ 10 & -20 & 10 \end{bmatrix}$$

Therefore $adjA = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

Now, $A^{-1} = \frac{1}{|A|}(adjA)$, $A^{-1} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$

So $X = A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 - 450 + 700 \\ 1800 + 0 - 1400 \\ -1200 + 900 + 700 \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix},$$

Hence $x = 5, y = 8, z =$

42. If $y = (tan^{-1}x)^2$ then show that $(x^2 + 1)^2 y_2 + 2x(x^2 + 1)y_1 = 2$

Ans:

$y = (tan^{-1}x)^2$ differentiate w.r.t x

$y_1 = 2tan^{-1}x \cdot \frac{1}{1+x^2}$ multiplying $(1 + x^2)$ on both side

$(1 + x^2)y_1 = 2tan^{-1}x$ again differentiate w.r.t x

$(1 + x^2)y_2 + y_1(2x) = 2 \cdot \frac{1}{1+x^2}$ multiplying $(1 + x^2)$ on both side

$(1 + x^2)[(1 + x^2)y_2 + y_1(2x)] = 2$

$(1 + x^2)^2 y_2 + (1 + x^2)y_1(2x) = 2$

$(x^2 + 1)^2 y_2 + 2x(1 + x^2)y_1 = 2$

43. Find the integral of $\frac{1}{x^2+a^2}$ with respect to x x and hence find $\int \frac{1}{x^2-6x+13}dx$

Ans: $\int \frac{1}{x^2+a^2}dx = \int \frac{asec^2\theta}{a^2tan^2\theta+a^2}d\theta$ put $x = atan\theta$ $\theta = tan^{-1}\left(\frac{x}{a}\right)$

$= \int \frac{asec^2\theta}{a^2(tan^2\theta+1)}d\theta$ $dx = asec^2\theta d\theta$

$= \int \frac{sec^2\theta}{asec^2\theta}d\theta$

$= \frac{1}{a}\int 1\, d\theta$

$= \frac{1}{a}\theta + C$

$= \frac{1}{a}tan^{-1}\left(\frac{x}{a}\right) + C$

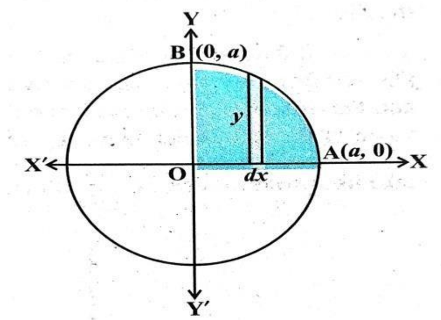
Therefore $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}tan^{-1}\left(\frac{x}{a}\right) + C$

Now, $\int \frac{1}{x^2-6x+13} dx = \int \frac{1}{x^2-2\cdot3\cdot x+9-9+13} dx$,   To make perfect square, add and subtract number

$= \int \frac{1}{(x-3)^2+2^2} dx = \frac{1}{2}tan^{-1}\left(\frac{x-3}{2}\right) + C$, $\left(\frac{b}{2\sqrt{a}}\right)^2 = \left(\frac{-6}{2\sqrt{1}}\right)^2 = 9$

44. Find the area of circle $x^2 + y^2 = a^2$ by the method of integration

Ans: $x^2 + y^2 = a^2$ we have $y = \sqrt{a^2 - x^2}$



Area of circle $= 4\int_0^a y\,dx$

$= 4\int_0^a \sqrt{a^2 - x^2}\,dx$

$= 4\left[\frac{x}{2}\cdot\sqrt{a^2 - x^2} + \frac{a^2}{2} sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$

$= 4\left[\left(0 + \frac{a^2}{2} sin^{-1}(1)\right) - \left(0 + \frac{a^2}{2} sin^{-1}(0)\right)\right]$

$= 4\left[\frac{a^2}{2}\cdot\frac{\pi}{2}\right]$

$= \pi a^2\ sq\ units$

45. Solve the differential equation $cos^2x\frac{dy}{dx} + y = tanx$, where $0 \le x \le \frac{\pi}{2}$

Ans: $cos^2x\frac{dy}{dx} + y = tanx$ dividing cos^2x on both side

$\frac{dy}{dx} + \frac{y}{cos^2x} = \frac{tanx}{cos^2x}$

$\frac{dy}{dx} + (sec^2x)y = tanx \cdot sec^2x$ Comparing with $\frac{dy}{dx} + Py = Q$

let $P = sec^2x$ and $Q = tanx \cdot sec^2x$

$I.F = e^{\int Pdx} = e^{\int sec^2xdx} = e^{tanx}$

Solution is $y \cdot (I.F) = \int Q \cdot (I.F)\,dx + C$

$y \cdot e^{tanx} = \int tanx \cdot sec^2x \cdot e^{tanx}\,dx$ put $t = tanx$

$y \cdot e^{tanx} = \int te^t\,dt$ Integrating by parts $dt = sec^2xdx$

$ye^{tanx} = t\int e^t\,dt - \int(\int e^t\,dt)1dt$

$ye^{tanx} = te^t - \int e^t\,dt$

$ye^{tanx} = te^t - e^t$

$ye^{tanx} = tanxe^{tanx} - e^{tanx} + C$

$ye^{tanx} = e^{tanx}(tanx - 1) + C$

PART-E

VI. Answer the following questions :

46. Prove that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and hence evaluate $\int_0^{\pi/4} \log(1 + tanx)\,dx$ **(6M)**

Ans: Let $t = a - x$ If $x = a$ $then$ $t = 0$ and $x = 0$ $then$ $t = a$

Then $x = a - t$

$dx = -dt$

Now $\int_0^a f(x)dx = -\int_a^0 f(a-t)dt$ w.k.t $\int_a^b f(x)dx = -\int_b^a f(x)dx$

$= \int_0^a f(a-t)dt$ w.k.t $\int_a^b f(x)dx = \int_a^b f(t)dt$

$= \int_0^a f(a-x)dx$

Therefore $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Now, $\int_0^{\pi/4} \log\left(1+tanx\right) dx$

$I = \int_0^{\frac{\pi}{4}} \log(1+tanx)\, dx$........................(1)

$I = \int_0^{\pi/4} \log\left(1+tan(\pi/4-x)\right) dx$ w.k.t $tan(\pi/4-x) = \frac{1-tanx}{1+tanx}$

$I = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-tanx}{1+tanx}\right) dx$

$I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+tanx}\right) dx$........................ (2)

Adding (1) and (2)

$I + I = \int_0^{\frac{\pi}{4}} \log(1+tanx)\, dx + \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+tanx}\right) dx$

$2I = \int_0^{\frac{\pi}{4}} \log\left[(1+tanx)\cdot\left(\frac{2}{1+tanx}\right)\right] dx$

$2I = \int_0^{\frac{\pi}{4}} \log 2\, dx$

$2I = \log 2 \int_0^{\frac{\pi}{4}} 1\, dx$

$2I = [\log 2\cdot x]_0^{\pi/4}$, $2I = log2\cdot\left[\frac{\pi}{4}-0\right]$, $I = \frac{\pi}{8} log2$

Therefore $\int_0^{\pi/4} \log\left(1+tanx\right) dx = \frac{\pi}{8} log2$

OR

Solve the following linear programming problem graphically: **(6M)**

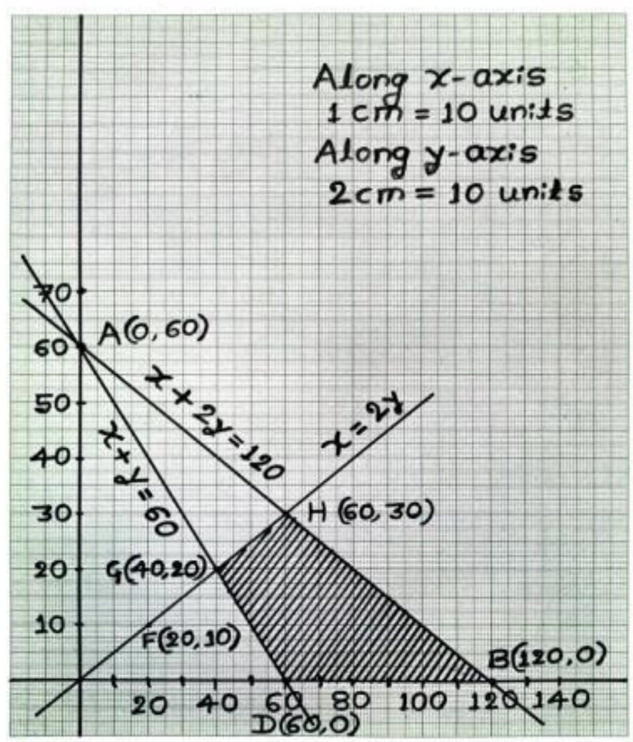
Minimise and Maximise $Z = 5x + 10y$

Subject $to\ x + 2y \le 120$

$x + y \ge 60$

$x - 2y \ge 0$

$x \ge 0, y \ge 0$



Ans: i) $x + 2y \le 120$

Put $x = 0$ and $y = 0$ ($0 \le 120$ True)

Equality form $x + 2y = 120$

	A	B
X	0	120
Y	60	0

ii) $x + y \ge 60$

Put $x = 0$ and $y = 0$ ($0 \ge 60$ False)

Equality form $x + y = 60$

	C	D
X	0	60
Y	60	0

ii) $x - 2y \ge 0$

$x \ge 2y$

Equality form $x = 2y$

	E	F	G
X	0	20	40
Y	0	10	20

Shaded region is the feasible region and it is bounded with corner points
D(60,0), B(120,0), H(60,30) and I(40,20)

Corner points	Value of Z $Z = 5x + 10y$
D(60,0)	**300**
B(120,0)	**600**
H(60,30)	**600**
I(40,20)	400

The maximum value of Z is 600 at corner point B(120,0) and H(60,30)
[The maximum value of Z is 600 at all the points on the line segment joining the points B(120,0) and H(60,30)]
The minimum value of Z is 300 at corner point D(60,0)

47. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$ and hence find A^{-1}

Ans: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Now, LHS $= A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = \text{RHS}$$

Using equation, to find inverse of matrix A

$$A^2 - 5A + 7I = O$$

$7I = 5A - A^2$ pre multiply A^{-1} on both side

$$7A^{-1}I = 5A^{-1}A - A^{-1}A^2$$

$$7A^{-1} = 5I - A$$

$$7A^{-1} = 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$7A^{-1} = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$7A^{-1} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix},$$

$$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

OR

Find the value of k if $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & if\ x \neq \pi/2 \\ 3 & if\ x = \pi/2 \end{cases}$ is continuous at $x = \pi/2$

Ans: $\lim\limits_{x\to\frac{\pi}{2}} f(x) = \lim\limits_{x\to\frac{\pi}{2}} \frac{k\cos x}{\pi - 2x}$ and $f\left(\frac{\pi}{2}\right) = 3$

$= \lim\limits_{x\to\frac{\pi}{2}} \frac{k\sin\left(\frac{\pi}{2}-x\right)}{2\left(\frac{\pi}{2}-x\right)}$ w.k.t $\cos x = \sin\left(\frac{\pi}{2} - x\right)$

$= \frac{k}{2}\lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}$ (1)

where $\theta =$ $\frac{}{2} - x$ if $x \to \frac{}{2}$ then $\theta \to 0$

Given that given function is continuous at $x = \frac{\pi}{2}$,

We have $\lim_{x \to \pi/2} f(x) = f\left(\frac{\pi}{2}\right)$

$$\frac{k}{2} = 3,$$
$$k = 6$$

PART-F

VII. For Visually Challenged Students Only

7. **If Statement 1: The left hand derivative of $f(x) = |x|$ at $x = 0$ is -1**
   Statement 2: The derivative of $f(x) = |x|$ exists at $x = 0$
   Then which of the following is true?
   a) Statement 1 is true, Statement 2 is false
   b) Statement 1 is false, Statement 2 is true
   c) Statements 1 and 2 Both are true
   d) Statements 1 and 2 Both are false

   Ans: a) Statement 1 is true, Statement 2 is false